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Analysis of a Waveguide Hybrid Junction by Rank Reduction

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Abstract—**Exact equations characterizing a waveguide hybrid junction traversed by a dielectric sheet are formulated by waveguide field-equivalence decomposition. A new reduced-rank spectral expansion technique avoids inversion of a large ill-conditioned matrix in the calculation of the scattering matrix. Arbitrary sheet thicknesses and permittivities are treated, accounting fully for waveguide boundaries and offset. For illustrative purposes, numerical results are presented for a rectangular waveguide hybrid, when only the dominant mode propagates.**

I. Introduction

THE INTEGRAL EQUATIONS, and the corresponding matrix equations, that represent scattering at a waveguide discontinuity often exhibit ill-conditioned behavior. In a previous paper [1] it was shown that the resultant difficulties can be largely overcome by taking advantage of the relatively low effective rank of the ill-conditioned portion of the matrix. In the following sections the new rank-reduction technique is applied to the waveguide hybrid junction problem. The hybrid junction, an important component of certain advanced microwave communication systems [2], [3] has resisted accurate analysis.

The hybrid junction to be analyzed is shown in Fig. 1. It consists of two crossed waveguides whose junction is traversed by a dielectric sheet at a 45° angle. By properly choosing the dielectric constant and sheet thickness, a directional coupler can be formed for a given frequency band. This coupler is used in band diplexing networks [4], [5]. An accurate analysis of the hybrid is important in the design of the band diplexer to achieve satisfactory frequency band separation and to identify spurious modes that may degrade system performance.

This paper will serve several purposes. It will illustrate the use of field superposition principles to decompose the complicated geometry of a hybrid junction into a combination of separate, uniform waveguides. It will discuss the application of this technique to derive an exact set of equations for the scattering matrix of the junction. It will further illustrate that, for those frequency intervals in which higher order modes affect the scattering significantly, their effect can be calculated despite the ill-conditioning of the equations, by use of a new rank-reduction technique. Finally, it will present the scattering coefficients for the hybrid, in a frequency range in which quasi-optical approximations are not valid.

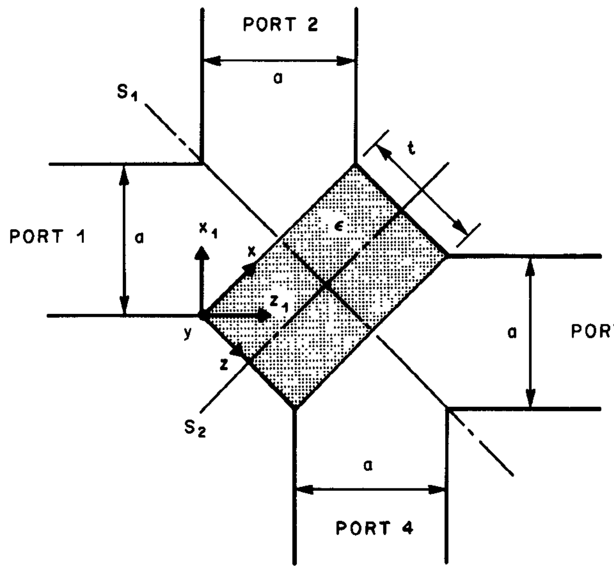

Fig. 1. Top view of a hybrid junction formed by two crossed waveguides of width a whose junction is traversed at a 45° angle by a dielectric sheet of thickness t and relative dielectric constant ε.

The analysis presented here is not restricted by the waveguide geometry, and permits the computation of higher order mode coupling. For purposes of illustration of the new technique, numerical results are presented for a rectangular waveguide hybrid over a frequency range in which only the

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dominant mode may propagate. The general analysis should be applicable to the multimode circular waveguide geometry of [3] and [4].

Almost all previous investigations of the hybrid junction have been made by quasi-optical approximations. It is commonly assumed that the waveguide is operated far enough above cutoff to allow the main mode and all spurious modes to be considered plane waves propagating in free space and obliquely incident upon the dielectric sheet [4], [6]. For such analyses to be practical, the sheet must be thin, to make the misaligned waveguide offset negligible. Even for a thin sheet, such solutions neglect the effects of the waveguide walls. Wardrop [7] improves upon the quasi-optical analysis by decomposing the incident mode into an angular spectrum of plane waves. Each of these plane waves is then considered to be in free space and is traced through to the other port apertures. At these output ports the plane waves are superimposed to yield the aperture field distribution and then the scattered mode levels. As applied, this method also largely ignores the waveguide boundaries. Unrau [8] approaches the problem by a mode-matching technique, using relatively complicated superpositions of modes in the field expansions. While the equations derived are exact, their solution requires the inversion of large ill-conditioned matrices. The analysis below is not restricted as to sheet thickness, dielectric constant, or frequency, accounts properly for the boundaries, and avoids ill-conditioned matrices.

To epitomize what follows, the solution of the hybrid junction is reduced to that of four simpler one-port networks by use of the symmetry properties of the hybrid. Each one-port structure is then made equivalent to two uniform waveguides with equivalent electric and magnetic current sheets. These structures are easier to analyze than the one-port structure from which they are derived. Integral equations for the current sheets are derived using the null-field condition in the two simpler waveguide structures. By writing series expansions for the current sheets, the integral equations are reduced to a system of linear algebraic equations for the current expansion coefficients. The equations are asymptotically ill-conditioned, yielding a large-order ill-conditioned matrix in the analysis. The inversion of this matrix is avoided by spectral decomposition and rank reduction, leading instead to inversion of a well-conditioned low-order matrix. This technique averts spurious resonances that arise from severe truncation of the matrix. The scattered fields are then readily determined from the approximate inversion.

II. Equivalent Problems

The determination of the scattering matrix reduces readily to the problem of a single mode incident only from port 1 of the hybrid junction. The fields scattered into all four ports are to be determined. By exciting all the ports with waves that preserve the symmetry of the structure with respect to planes S_1,S_2, the original structure can be made equivalent to the superposition of four 4-port structures in which S_1 and S_2 are equivalent to electric or magnetic walls.

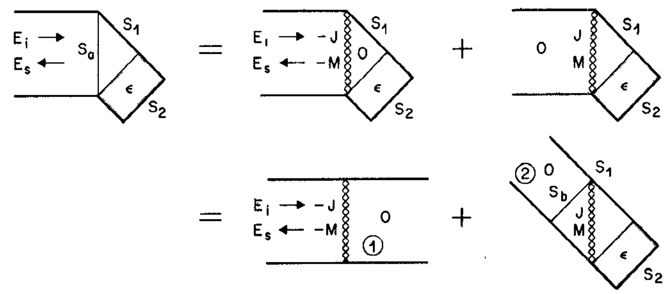

Fig. 2. Equivalence-theorem decomposition into two waveguides, each having a null-field region (0). The null-field media are then altered to yield two simple waveguide structures.

TABLE I
Types of Equivalent Boundary Surfaces for Combinations of Symmetrical Excitations

	Sign of Excitation				Boundary Surface	
Port	1	2	3	4	S_1	S_2
	+	-	+	-	E	E
	+	-	-	+	E	M
	+	+	+	+	M	M
	+	+	-	-	M	E

E = electric wall, M = magnetic wall

Table I gives the four required sets of excitations and the resulting type of symmetry-plane surface for each. The original four-port structure is thereby reduced to four uncoupled one-port structures. These are all identical and only the single one-port structure shown at the left in Fig. 2 needs to be analyzed. By using equivalent current sheets at the discontinuity surface S_a, the one-port is made equivalent to the superposition of two waveguides, each excited by a current sheet and exhibiting a null-field region on one side of the current sheet surface [9]. The medium in each null-field region is thereby made free to be altered without changing the fields elsewhere, allowing reduction to two uniform waveguide structures excited by current sheets. The equivalent current distributions are to be found, by requiring them to produce the assumed null fields.

III. Analysis of Equivalent One-Port Structure

The two simple waveguide structures obtained by the decompositions shown in Fig. 2 are almost in the form of the two simple waveguide problems analyzed in the previous paper [1]. A nearly identical treatment applies, so that only the salient features are repeated here, with the differences noted below.

The Lorentz Reciprocity Theorem is used to express the amplitude of each mode radiated by the combination of the incident mode and the as yet unknown equivalent current sheets. If the currents were known this would yield the scattered mode amplitudes. The currents can be determined by setting the coefficients of each mode equal to zero at surfaces S_a and S_b, which suffices to annihilate the fields throughout the null-field regions. This results in integral equations for the unknown electric and magnetic currents $\boldsymbol{J}$ and $\boldsymbol{M}$. A set of linear algebraic equations is then obtained by substituting a suitable series expansion for each current in the integral equations. This exact system of equations for the unknown current coefficient vector c in terms of the known incident modal amplitude vector s can be written in matrix form as

$$Gc = s. \tag{1}$$

The infinite-order matrix G can be partitioned as

$$G = \begin{bmatrix} G_{1e} & G_{1h} \\ G_{2e} & G_{2h} \end{bmatrix} \tag{2}$$

where the submatrix elements are

$$G_{1e}(m,n) = -(1/P_{1m}) \int_{S_a} \boldsymbol{E}_{1m}^{-} \cdot \boldsymbol{J}_n \, dS \tag{3}$$

$$G_{1h}(m,n) = (1/P_{1m}) \int_{S_a} \boldsymbol{H}_{1m}^{-} \cdot \boldsymbol{M}_n \, dS \tag{4}$$

$$G_{2e}(m,n) = (1/P_{2m}) \left[\int_{S_a} \boldsymbol{E}_{2m}^{+} \cdot \boldsymbol{J}_n \, dS + R_m \int_{S_a} \boldsymbol{E}_{2m}^{-} \cdot \boldsymbol{J}_n \, dS \right] \tag{5}$$

$$G_{2h}(m,n) = -(1/P_{2m}) \left[\int_{S_a} \boldsymbol{H}_{2m}^{+} \cdot \boldsymbol{M}_n \, dS + R_m \int_{S_a} \boldsymbol{H}_{2m}^{-} \cdot \boldsymbol{M}_n \, dS \right]. \tag{6}$$

The integrals involve scalar products of normal modes of the uniform waveguide regions 1 and 2, shown in Fig. 2, with the known current expansion functions, evaluated on the surface S_a. P_{1m} and P_{2m} are known modal normalization factors for the two waveguides, and R_m is just the reflection coefficient of the mth mode when incident upon the dielectric region terminated by either the electric wall or the magnetic wall at S_2.

To illustrate the approach used in solving (1) for the current c, assume a rectangular waveguide geometry for the hybrid junction of Fig. 1. Also assume that only TE_{n0} modes are incident. Then since the structure has no y variation, only TE_{n0} modes will be scattered. The electric field is along $\hat{y}$, and, therefore, $\boldsymbol{M}$, which is confined to the discontinuity surface, is along $\hat{x}_1$. The magnetic field is in the plane orthogonal to $\hat{y}$, and, therefore, $\boldsymbol{J}$, which also exists only on the discontinuity surface, is along $\hat{y}$. The amplitude distributions of these currents are expressed as infinite series expansions. The convergence of the algebraic equations will be dependent upon the choice of expansion functions. Since $\boldsymbol{J}$ and $\boldsymbol{M}$ are related to the discontinuities in the tangential field component, it seems reasonable to choose the normal modes of waveguide region 2 as the current expansion functions. The elements of G can then be evaluated. The direct inversion of G is not practical, even when truncated, because it is ill-conditioned [1]. However, a low-rank spectral decomposition of G can be made to avoid inversion of a large-order matrix.

As in the previous paper [1], partition the matrix equation so as to rewrite $Gc = s$ as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}. \tag{7}$$

The square matrix A is what G is truncated to if only the first few terms in the expansions for $\boldsymbol{J}$ and $\boldsymbol{M}$ are kept and only the first few modes of waveguides 1 and 2 are required to satisfy the null-field condition at planes S_a and S_b. The vector of the current expansion coefficients c is partitioned into p, containing the first few coefficients, and q, for the higher order ones. The known incident modal amplitude vector s is partitioned as r and the null vector since, for a single incident mode, s and r have only one nonzero element. For more general excitations, s may be partitioned into two nonzero parts and the solution readily extended to include the additional source.

If G is severely truncated to merely A, a crude solution p' for the first few unknown current expansion coefficients p is obtained as

$$p' = A^{-1}r. \tag{8}$$

If G were not truncated, the exact solution for all the unknown current expansion coefficients p,q would be obtainable from the correction to (8) given by

$$p = (A - BD^{-1}C)^{-1}r \tag{9}$$

$$q = -D^{-1}Cp. \tag{10}$$

These, however, entail the direct inversion of the ill-conditioned matrix D, an impractical procedure. However, if, as is often at least approximately the case, the rank K of matrix D is effectively small, a useful alternate expression for D may be written as

$$D = fg \tag{11}$$

where f and g are formed, for a diagonalizable matrix, from the K dominant right and left eigenvectors of D, or else from its singular-value decomposition, retaining only the K dominant singular values [10]. Matrices f and g are $N \times K$ and $K \times N$, respectively, when D is, for practical purposes, truncated to a large but finite size $N \times N$, with N usually much larger than K. Then an approximate solution for the current coefficients, more accurate than (8) in that it does not ignore the effects of higher order modes and expansion functions, is given by

$$p = (A - BD^{-}C)^{-1}r \tag{12}$$

$$q \doteq -D^{-}Cp. \tag{13}$$

Here, D^{-} is the group inverse of D, given by $D^{-} = f(gf)^{-2}g$. The utility of the method stems from the fact that the largest matrix to be inverted is gf, which is only $K \times K$ and not ill-conditioned. If, as in [1], the equation $Cp + Dq = 0$ is premultiplied by the Hermitian conjugate D^{+} of D and the same process is applied, then the group inverse of $D^{+}D$ yields the minimum-norm least squares solution for q in terms of p [10]. The accuracy of this approach will be discussed after some results are presented.

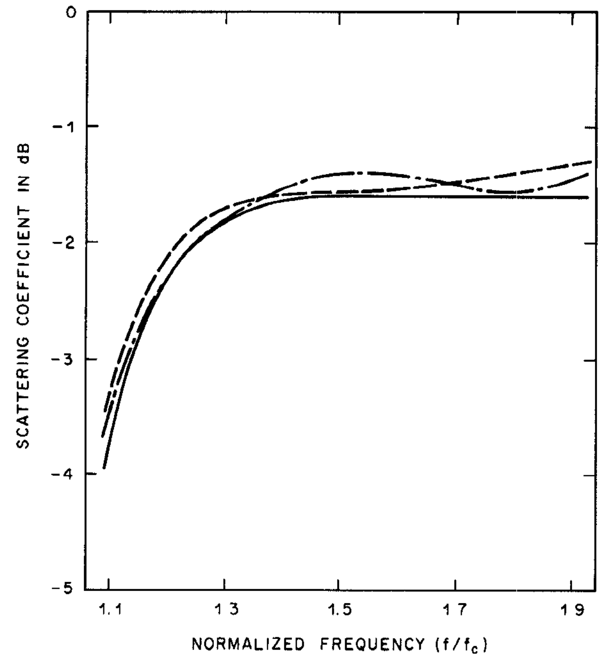

Fig. 3. Transmission coefficient from port 1 to port 3 for the empty junction. The solid line is for the severely truncated matrix method, the broken line shows the effect of higher order terms using rank reduction, and the dashed line is the exact result given by Bouwkamp [11].

IV. Numerical Results

The technique described in the preceding sections was used to obtain the scattered fields for the rectangular waveguide hybrid junction of Fig. 1. A TE_{10} mode is assumed to be incident from port 1. Results are presented here for dominant mode operation, although the analysis is not restricted to this frequency range. While higher order modes do not propagate in this frequency range, coupling to these modes is included in the equations and could readily be computed when there is multimode operation. Three cases are presented: a zero-thickness sheet with unity as the relative dielectric constant (i.e., no dielectric at all and no offset in waveguide alignment); a one-quarter guide-wavelength thick sheet at the design frequency $f/f_c = 1.5$, with a relative dielectric constant of five; and a half guide-wavelength thick sheet at $f/f_c = 1.5$, also with a relative dielectric constant of five. The frequency normalization is carried out with respect to the TE_{10} cutoff frequency f_c in the rectangular guide of width a.

The effects of the higher order modes in the hybrid were found to vary with frequency. Over some frequency ranges the corrections introduced by use of the group inverse of D were negligible, but, near certain frequencies, resonance-like peaks that could not be ascribed to any physical or geometrical causes appeared when the corrections were ignored. At such frequencies, the truncated matrix A itself exhibited singular behavior. The resulting difficulties were readily overcome, however, by applying the rank-reduction technique directly to this matrix, completely eliminating the spurious resonances.

The empty junction case, without offset, has already been solved exactly by Bouwkamp [11]. Fig. 3, which shows the

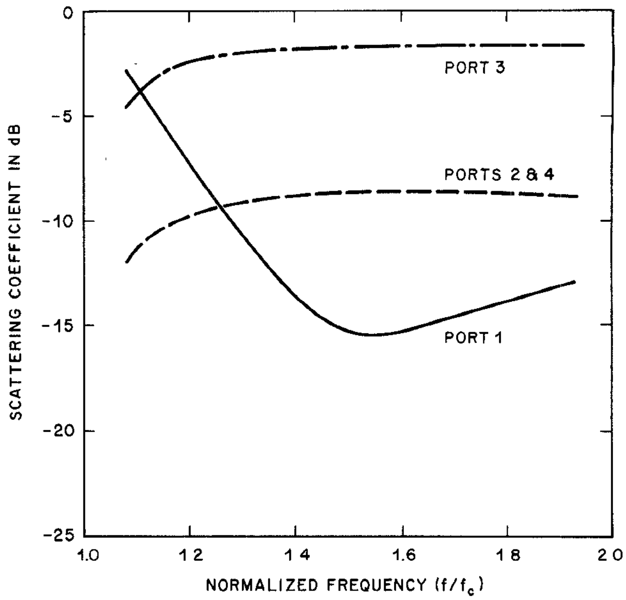

Fig. 4. Scattering coefficients for an empty junction (zero-thickness sheet with a relative dielectric constant of one).

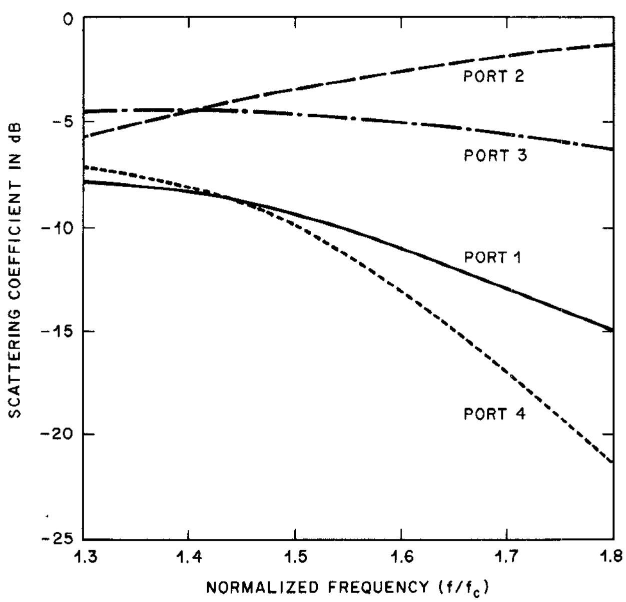

Fig. 5. Scattering coefficients for a sheet a quarter-wave thick at $f/f_c = 1.5$ with relative dielectric constant $\varepsilon = 5$.

transmission coefficient from port 1 to port 3, is illustrative of the agreement (to within a fraction of a decibel) with Bouwkamp, achievable with only two terms used in the expansion for the electric and magnetic current sheets. The severely truncated matrix A is then only 4×4. The figure also shows the small effect of the higher order terms upon the severely truncated solution. In this case, D^+D was taken to be 8×8 and was approximately of rank two. Similar results are obtained for the scattering into the other three ports.

The scattered fields are presented in Figs. 4–6 for the cases of zero thickness, quarter-wave thickness, and half-wave thickness. The truncated equations, with A only 4×4, proved adequate for most frequencies but corrections for

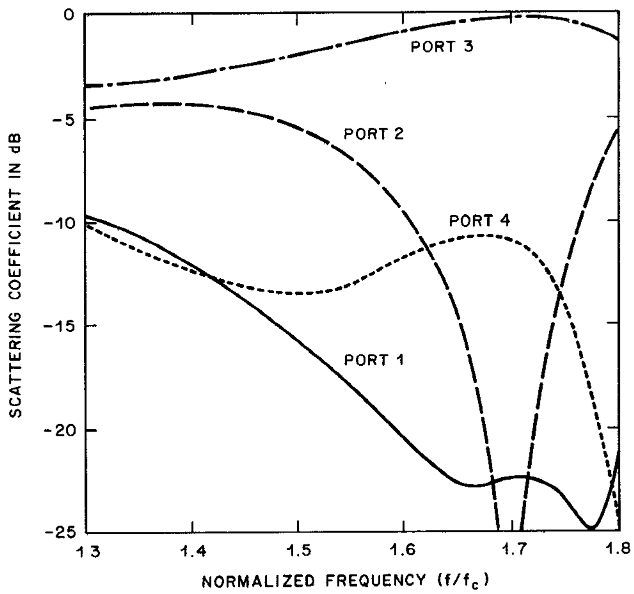

Fig. 6. Scattering coefficients for a half-wave-thick sheet (at $f/f_c = 1.5$) with relative dielectric constant $\varepsilon = 5$.

higher order terms were needed near certain spurious resonant frequencies. Rank reduction provided the corrections in those intervals. While the zero-thickness case has been solved exactly before [11], the finite-thickness cases have not.

It is instructive to compare the thick-sheet results of Figs. 5 and 6 with what might be predicted using the quasi-optical approximation that replaces the TE_{10} mode with a plane wave incident at a 45° angle upon an infinite dielectric sheet in free space, with no boundaries. This approximation is inappropriate and not intended for the dominant mode regime, but there are no better solutions available for comparison. Relying on the unbounded plane wave model suggests that the signals scattered into ports 2 and 3 should be 2.0 and 4.4 dB down, respectively, from the incident signal and that no signal should appear in ports 1 and 4, near $f/f_c = 1.55$. From Fig. 5, the signals at ports 2 and 3 are, respectively, 3.0 and 4.8 dB down at this frequency. There is also some energy scattered into ports 1 and 4. For the half guide-wavelength case, the total transmission from port 1 to port 3 is predicted quasi-optically at $f/f_c = 1.55$. From Fig. 6 it is seen that the transmission peak is at the higher frequency $f/f_c = 1.7$. While very little energy is then reflected into port 2, there is some loss of signal into ports 1 and 4 at this frequency.

The accuracy or reliability of the solution process is not predictable in general but is subject to certain numerical checks. Besides monitoring how well power conservation and reciprocity are satisfied, the convergence of the results with increasing values of the reduced rank K and with an increasing order of the truncated matrix A can be and has been verified. The examples cited were also compared with the solutions obtainable by direct inversion of large-order versions of the G matrix. Typically, a rank-reduced 4×4 inversion agreed with the direct inversion of a 12×12 matrix to within a few percent. It should be noted that, in accordance with the variational principle that governs the scattering calculations [12], the results for the reflection and transmission coefficients are more accurate than those for the equivalent current sources.

Formally, the errors in the approximations for p and q involve the matrix $(I - DD^-)C$ as a factor. While the norm of this expression need not be small for arbitrary systems of linear equations, the resultant errors in the current can be expected to be small for waveguide junction problems. This behavior can be traced to that of the integrals that form the elements of the G matrix. The D matrix is the asymptotic form, for large row and column numbers, of the G matrix and B and C are similarly asymptotic to D for large column and row numbers, respectively. Consequently, the spaces spanned by the reduced-rank D matrix and by B or C are not expected to differ much asymptotically, so that the error, which represents the mismatch in these spaces, becomes small in such problems.

V. Conclusions

Scattering by a waveguide hybrid junction can be analyzed accurately by using waveguide field equivalence theorems to derive null-field equations for equivalent current sheets, from which the scattered fields can readily be computed. The resulting exact equations for the hybrid junction are of infinite order and asymptotically ill-conditioned and thus difficult or impractical to solve by ordinary techniques. However, the inversion of a large-order ill-conditioned matrix is avoided by using the rank-reduction technique [1], which permits a useful solution to the large set of ill-conditioned equations by direct inversion of only a low-order well-conditioned matrix. Spurious resonances, caused by a nearly singular truncated matrix, were eliminated by this technique. Numerical results have been presented for the rectangular waveguide hybrid junction in the dominant mode regime. The zero-thickness case was found to be in close agreement with previously published exact results [11]. Two other cases, those of quarter-wave- and half-wave-thick sheets with relative dielectric constants of five, were also analyzed. While the computations presented here were for a rectangular waveguide geometry in the dominant mode frequency regime, these are not limitations of the analysis. By using modes and current expansion functions appropriate for a circular waveguide operated in the multimode regime, numerical results would be obtainable for the hybrids of [3] and [4]. The hybrid junction considered here has a single layer, but the analysis is easily extended to the multilayer hybrid, merely by using the properly defined reflection coefficient in (5) and (6). The approach is not restricted as to sheet thickness, dielectric constant, or frequency and does not neglect the waveguide boundaries or the waveguide offset at opposite ports when the dielectric sheet is thick.

Acknowledgment

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Propagation Losses of Guided Modes in an Optical Graded-Index Slab Waveguide with Metal Cladding

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***Abstract*—Analytical results for propagation losses of guided modes in a graded-index slab waveguide (GISW) with metal cladding are presented. When the permittivity in the guiding layer decreases linearly away from the metal surface, the attenuation constant α_G of well-guided modes, TE and TM, is approximately proportional to only the ratio $(\Delta\varepsilon_i/\varepsilon_0)/d_i$, where $\Delta\varepsilon_i$ is the increment in the permittivity at the metal surface in the direction of the polarization of optical waves, d_i is the diffusion depth in this direction, and ε_0 is the permittivity of free space.**

I. Introduction

ELECTROOPTIC crystal, such as $LiNbO_3$ or $LiTaO_3$, is very promising for use as the substrate of an integrated optical circuit. Recently, several experiments have been reported on the techniques for fabricating optical guides in these crystals, which consist of diffusing suitable metal ions into the substrate [1], [2] and out-diffusing Li_2O from the surface [3]. These methods yield a graded-index slab waveguide (GISW) instead of a step-index slab waveguide (SISW) in which the guiding layer has the space-invariant permittivity. Analyses of guided modes in the GISW have been made for the exponential [4] and linear [5] profiles of the permittivity. To interact the guided modes with low-frequency electromagnetic fields, metal electrodes with planar structures are generally needed. The GISW should become lossy due to the metal cladding on the surface. The effects of metal cladding have been examined for the SISW in homogeneous media in order to form a mode filter and an optical strip line [6]–[8]. On the other hand, the guiding properties of a metal-clad GISW, especially propagation losses of guided modes, have not been elucidated.

In this paper, propagation losses of guided modes in the GISW with metal cladding are analyzed under the assumption that the permittivity in the guiding layer decreases linearly away from the metal surface, taking the anisotropy of the electrooptic crystal into account. The effective thickness of the guiding layer is derived approximately. The attenuation constant α_G of guided modes in a metal-clad GISW is estimated in comparison with the attenuation constant α_S of guided modes in a metal-clad SISW.

Numerical solutions of the dispersion equation are given for both TE and TM modes in c-plate $LiNbO_3$.

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II. Analysis

A. Derivation of the Dispersion Equation

In our analysis, we assume that the optical wave varies as $\exp j(\omega t - k_z z)$ and that all quantities are independent of x,